Does Higher Uncertainty Cause Recessions?

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Abstract

This paper readdresses the question of whether temporarily higher uncertainty can lead to recessions. In particular, it is studied whether the dynamics induced by uncertainty shocks differ depending on whether they apply to demand conditions or to TFP. To answer the research question a partial equilibrium model that features heterogeneous firms, uncertainty shocks and various forms of capital adjustment costs is built and simulated. The central finding of the paper is that while uncertainty shocks to demand cause the bust, rebound and overshoot dynamics reminiscent of recessions, uncertainty shocks to TFP are likely to lead to large and persistent booms. This result can be easily understood when considering that uncertainty shocks in the model have an expectational effect as well as a distributional effect. While the expectational effect is negative in the presence of non-convex adjustment costs, its magnitude does not change much between uncertainty shocks to demand and TFP. In contrast to this, the distributional effect is positive and an order of magnitude larger for uncertainty shocks to TFP than for uncertainty shocks to demand. The intuition is that for TFP shocks the revenue function is likely to have increasing returns to scale while for demand shocks the returns to scale can be constant at best. Hence, for TFP shocks higher ex-post cross-sectional dispersion is a time of opportunity which more than compensates for the negative expectational effect of uncertainty shocks, causing a prolonged boom in aggregates. For uncertainty shocks to demand the negative expectational effect dominates the distributional effect causing the recession like dynamics emphasized by Bloom (2009).

Keywords: Uncertainty Shocks, Investment, Adjustment Costs, Heterogeneous Firms, Business Cycles, Demand vs. Supply

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1 Introduction

Ever since the outbreak of the financial crises in 2008, the use of the word uncertainty in relation to macroeconomic events has become increasingly popular among policy makers and the media. Statements such as ‘uncertainty affects behaviour, which feeds the crisis’\(^1\) by Olivier Blanchard and headlines such as ‘Economic uncertainty drags US retail sales’\(^2\) illustrate the widely shared view that heightened uncertainty is at least partly responsible for low economic activity. The idea that uncertainty is an important factor for driving aggregate economic outcomes also has a long tradition in economics, dating back at least to Knight (1971) and Keynes (1936). However, even though the role of uncertainty has been a major research area in the investment literature ever since the seminal paper by Bernanke (1983), the modern quantitative business cycle literature has generally abstracted from variations in uncertainty as impulses driving aggregate fluctuations.\(^3\)

Since the publication of a recent paper by Bloom (2009), the academic interest in considering variations in uncertainty as impulses driving the business cycle has been revived again.\(^4\) In a partial equilibrium heterogeneous firm model that features various forms of labor and capital adjustment costs, he shows quantitatively that a temporary increase in uncertainty first leads to a drop and then a subsequent rebound and overshoot of output and employment. In the paper variations in uncertainty are modeled as changes in the variance of idiosyncratic and aggregate shocks that hit the firms. This approach of modeling variations in uncertainty has since then been applied in general equilibrium models with adjustment costs by Bloom et al. (2010) and Bachmann and Bayer (2011). While the former paper confirms that the drop, rebound and overshoot dynamics are robust to general equilibrium considerations the latter paper argues that uncertainty shocks are unlikely to be a major quantitative source of business cycle fluctuations. One potential reason for these discrepancies in findings is that Bloom (2009) implicitly considers uncertainty shocks to demand, while Bachmann and Bayer (2011) consider uncertainty shocks to total factor productivity (TFP).

With this background in mind, the current paper readdresses the question of whether temporarily higher uncertainty can lead to recessions. In particular, it is studied whether the dynamics induced by uncertainty shocks differ depending on whether they apply to demand conditions or to TFP. To answer the research question a partial equilibrium model that features heterogeneous firms, uncertainty shocks and various forms of capital adjustment costs is built and simulated. In line with the existing literature, uncertainty shocks are modeled as changes in the variance of idiosyncratic profitability shocks. The main difference in the model set-up compared to the literature is that the specification of the revenue function allows to separately analyze the quantitative implications of uncertainty shocks to demand and TFP.

The central finding of the paper is that while uncertainty shocks to demand cause

\(^1\)Quote from guest article by Olivier Blanchard in the *Economist* on January 29, 2009.

\(^2\)Article headline in the *Financial Times* on June 2, 2011.

\(^3\)Other papers that consider the effect of uncertainty on investment include Hartman (1972), Abel (1983), Caballero (1991), Dixit and Pindyck (1994), Leahy and Whited (1996) and Guiso and Parigi (1999) among others.

\(^4\)A discussion of the related literature can be found towards the end of the introduction.
the bust, rebound and overshoot dynamics reminiscent of recessions, uncertainty shocks to TFP are likely to lead to large and persistent booms. The difference in these dynamic effects of uncertainty shocks is caused by different degrees of returns to scale in the revenue function that are implied by demand and TFP shocks. This result can be easily understood when considering that uncertainty shocks in the model have an expectational effect as well as a distributional effect: On the one hand, expectations about the future get more uncertain and on the other hand the cross-sectional dispersion of profitability across firms increases after a positive uncertainty shock.

While the expectational effect is negative in the presence of non-convex adjustment costs, its magnitude does not change much between uncertainty shocks to demand and TFP. In contrast to this, the distributional effect is positive and is an order of magnitude larger for uncertainty shocks to TFP than for uncertainty shocks to demand. The intuition is that for TFP shocks the revenue function is likely to have increasing returns to scale while for demand shocks the returns to scale can be constant at best. Hence, for TFP shocks higher ex-post cross-sectional dispersion is a time of opportunity which more than compensates for the negative expectational effect of uncertainty shocks, causing a prolonged boom in aggregates. For uncertainty shocks to demand the negative expectational effect dominates the distributional effect causing the recession like dynamics emphasized by Bloom (2009).

A secondary contribution of the paper is that it is shown that uncertainty shocks have first moment implications for the distribution of profitability whenever the driving process is specified as an AR(1) process in logs, which is a common assumption in the literature. The reason for this is that an increase in the variance of a log-normal variable actually translates into an increase in the mean of the variable in levels. Adjusting for this positive first moment effect of uncertainty shocks is not easy as long as the persistence parameter of the AR(1) process is not equal to zero or one. This fact needs to be taken into account when building quantitative models that include uncertainty shocks.

There are various strands of literature that are related to this paper. Most relevant is the recent quantitative literature on the impact of uncertainty shocks that was started by Bloom (2009). Other papers that consider uncertainty shocks in models with capital adjustment costs are Bloom et al. (2007), Bloom et al. (2010) and Bachmann and Bayer (2011). Moreover, there are various recent papers on the interaction of time-varying uncertainty with financial frictions as for example Dorofeenko et al. (2008), Gilchrist et al. (2009), Chugh (2010) and Arellano et al. (2011). The quantitative literature that was started by Bloom (2009) builds itself on an extensive literature that considers the effect of uncertainty on investment. Important contributions in this line of research include Hartman (1972), Abel (1983), Bernanke (1983), Caballero (1991) and Dixit and Pindyck (1994). Finally, this paper builds on many papers that study the effects of capital adjustment costs such as Hayashi (1982), Abel and Eberly (1994), Abel and Eberly (1996), Caballero and Engel (1999) and Cooper and Haltiwanger (2006) among others.

The rest of the paper proceeds along the following lines. In the next section the model set-up is presented and some necessary concepts for the analysis of uncertainty shocks are formalized. In section three analytic results are derived for the model
without any capital adjustment costs. This case helps to understand the underlying
dynamics in the model and in addition serves as a benchmark to which we can
compare the results for models that incorporate various forms of capital adjustment
costs. Section four then moves on to analyze various models with capital adjustment
costs making use of numerical simulations. Finally, section five provides a brief
conclusion of the paper. All derivations of the analytical results are contained in
the Appendix.

2 A Firm Model with Adjustment Costs and Uncertainty Shocks

In this section a partial equilibrium model with heterogeneous firms is presented. As in Abel and Eberly (1994) and Cooper and Haltiwanger (2006) the firms are assumed to face a rich set of convex and non-convex adjustment costs to investment, as well as partial investment irreversibilities. Furthermore, in line with the recent uncertainty shocks literature that was started by the paper of Bloom (2009), the model features variations in uncertainty over time, which are modeled as changes in the variance of idiosyncratic profitability shocks faced by each firm. Labor adjustment costs are disregarded to keep the model as simple as possible, while allowing for an analysis of the interaction between variations in uncertainty and aggregate investment, employment and output. The main difference in the model set-up compared to the existing literature is that the specification of the revenue function allows to separately analyze the effect of uncertainty shocks to demand (Demand shifter) and supply (TFP).

This set-up is mainly motivated by the fact that the existing literature has made varying implicit assumptions about the underlying structural nature of the uncertainty shocks. This might be a possible reason for why Bloom (2009) finds a considerable negative impact of uncertainty shocks, while Bachmann and Bayer (2011) find that uncertainty shocks do not alter business cycle properties a lot. The former paper implicitly assumes uncertainty shocks to demand, while the latter assumes that they are uncertainty shocks to TFP. As will be shown below, the common way that uncertainty shocks are modeled implies that on the one hand expectations about the future get more uncertain upon impact and on the other hand the cross-

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5Throughout the paper the terms firm and plant are used interchangeably, as it is assumed that each firm just operates one plant.

6See the introduction for a complete list of papers that model uncertainty shocks in this way.

7This approach can be justified on the grounds that Bloom et al. (2010) show that uncertainty shocks lead to a drop and rebound in output even when the model only features capital adjustment costs. Moreover, Bloom (2009) shows that disregarding labor adjustment costs affects the fit of his model by an order of magnitude less compared to disregarding capital adjustment costs.

8In the literature it is common to work with a reduced form shock to either revenues or profits that could be due to demand, supply (TFP) or wage movements. The flexible way the revenue function is specified in this paper allows to study these effects separately.

9As will be argued below, the admissible returns to scale of the revenue function are to a large extent determined by the type of structural shock we look at. Hence, by making assumptions about the returns to scale of the revenue or profit function, we implicitly say something about the underlying shock that is assumed.
sectional dispersion of profitability across firms increases after a positive uncertainty shock. The former channel will be labeled as the expectational effect and the latter as the distributional effect.\textsuperscript{10} In the remainder of the paper it will be shown that the magnitude of the distributional effect depends to a large extent on the structural nature of the uncertainty shock. Uncertainty shocks to TFP have a much higher positive distributional effect than uncertainty shocks to demand. In contrast, the expectational effect is negative and similar in magnitude for both types of uncertainty shocks in the presence of non-convex adjustment costs. Uncertainty shocks to demand therefore lead to the bust, rebound and overshoot dynamics stressed in the literature, while uncertainty shocks to supply lead to considerable booms.

\section{Production, Demand and the Revenue Function}

There is a continuum of risk neutral firms indexed by \( i \in [0,1] \) who maximize the present discounted value of expected profit streams. Risk neutrality is assumed to isolate the effect of time-varying uncertainty that arises through the presence of non-convex adjustment costs which induce real options effects. Firms are assumed to face a constant or decreasing returns to scale (RTS) production function and a constant elasticity demand function.\textsuperscript{11} Moreover, it is assumed that firms differ in their productivity and potentially in the demand that they face. Under these assumptions a revenue function with the following properties can be derived as shown in Appendix B:\textsuperscript{12}

\[
S(A_{i,t}, K_{i,t}, L_{i,t}) = A_{i,t}^c K_{i,t}^a L_{i,t}^b
\]  

(1)

In the above equation, \( A_{i,t}^c = B_{i,t}^{1/\varepsilon} \tilde{A}_{i,t}^{(\varepsilon-1)/\varepsilon} \) is a reduced form shock that is comprised of TFP shocks (\( \tilde{A} \)) and shocks to the demand shifter of the constant elasticity demand function (\( B_{i,t} \)). In addition, \( K \) and \( L \) represent the capital stock and labor input, while the exponents \( a, b, \) and \( c \) determine the curvature and RTS of the revenue function. In particular, it is always the case that both \( a, b \in (0,1) \) and in addition \( a + b < 1 \). Hence, the revenue function always displays decreasing returns to scale in \((K, L)\) space. This is due to either the assumption of decreasing RTS in the production function and/or some degree of market power. However, the exact values of \( a \) and \( b \) will be determined by the specific assumptions about the demand elasticity \( \varepsilon \) and RTS of the production function.\textsuperscript{13}

Moreover, the value of the exponent on the profitability shock \( c \) will depend on whether we look at demand or TFP shocks. In particular, for demand shocks

\textsuperscript{10}Bloom (2009) refers to these two channels as the uncertainty and the volatility effect. As both are induced by the uncertainty shock it is deemed that the distinction between an expectational effect and a distributional effect seems clearer.

\textsuperscript{11}The production function takes the following form \( Y(\tilde{A}_{i,t}, K_{i,t}, L_{i,t}) = \tilde{A}_{i,t} K_{i,t}^\nu L_{i,t}^\omega \) while the demand function takes the form \( q_{i,t} = B_{i,t} p_{i,t}^{-\varepsilon} \).

\textsuperscript{12}The derivation of this revenue function is closely related to the one in Bloom (2009) with the difference that in his model he assumes \( c = 1 - a - b \), while the specification in this paper allows for flexible values of \( c \). This will be of importance in the analysis further below.

\textsuperscript{13}This connection becomes clear when looking at the formulas for those exponents: \( a = \nu \frac{\varepsilon-1}{\varepsilon} \) and \( b = \omega \frac{\varepsilon-1}{\varepsilon} \).
\( c = 1/\varepsilon \in (0, 1) \) and in the case of supply shocks \( c = (\varepsilon - 1)/\varepsilon \in (0, 1) \). Furthermore, the revenue function can only have constant or decreasing RTS in \((A, K, L)\) space for demand shocks\(^{14}\) while the revenue function can have either of decreasing, constant or increasing RTS for TFP shocks.\(^{15}\) However, a moderate elasticity of demand will usually imply increasing returns to scale in \((A, K, L)\) space and a value of \( c \) that is close to but below one. The RTS of the revenue function will be especially important in determining how the distributional effect of uncertainty shocks impacts on aggregate investment, employment and revenues, which will be shown in section 3.1.

At this point it is worth to relate the specification of the revenue function in equation (1) to other papers on investment under uncertainty. For example Bloom (2009) works with a specification of \( c = 1 - a - b \). Given his assumptions of constant returns to scale in production and some degree of market power, this implies that his shocks to \( A \) are demand shocks. In contrast, Bloom et al. (2010) and Bachmann and Bayer (2011) use a model with perfect competition so that the revenue function is equal to the assumed decreasing returns to scale production function. Their specification would therefore be equivalent to setting \( c = 1 \) and interpreting \( A \) as TFP shocks. Finally, Cooper and Haltiwanger (2006) and Caballero and Engel (1999) work with a reduced form profit function that would imply a value of \( c = 1 - b \) if the current economic structure was assumed.\(^{16}\) These seemingly minor assumptions will turn out to be important determinants for the distributional effect of uncertainty shocks. Table 1 summarizes the values of \( c \) that are commonly made in the investment literature along with some other common modeling assumptions that will be used further below.

Table 1: Common modeling assumptions in the investment literature

<table>
<thead>
<tr>
<th>Paper</th>
<th>Exponent on A</th>
<th>( Z \cdot \Psi )</th>
<th>Form of process</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooper and Haltiwanger (2006)</td>
<td>( c = 1 - b )</td>
<td>Yes</td>
<td>AR(1) in logs</td>
</tr>
<tr>
<td>Bloom (2009)</td>
<td>( c = 1 - a - b )</td>
<td>Yes</td>
<td>Geometric RW</td>
</tr>
<tr>
<td>Bloom et al. (2010)</td>
<td>( c = 1 )</td>
<td>Yes</td>
<td>AR(1) in logs</td>
</tr>
<tr>
<td>Bachmann and Bayer (2011)</td>
<td>( c = 1 )</td>
<td>Yes</td>
<td>AR(1) in logs</td>
</tr>
<tr>
<td>Khan and Thomas (2008)</td>
<td>( c = 1 )</td>
<td>Yes</td>
<td>Markov chain</td>
</tr>
<tr>
<td>Thomas (2002)</td>
<td>( c = 1 )</td>
<td>Only ( Z )</td>
<td>AR(1) in logs</td>
</tr>
<tr>
<td>Caballero and Engel (1999)</td>
<td>( c = 1 - b )</td>
<td>Only ( Z )</td>
<td>Geometric RW</td>
</tr>
</tbody>
</table>

\(^{14}\)This is easy to see by noting that in this case \( a + b + c = [1 + (\nu + \omega)(\varepsilon - 1)]/\varepsilon \leq 1 \) due to the fact that \( \nu + \omega \in (0, 1] \).

\(^{15}\)Here we have that \( a + b + c = (\varepsilon - 1 + (\nu + \omega)(\varepsilon - 1))/\varepsilon \leq 2(\varepsilon - 1)/\varepsilon \). Hence, whenever the demand elasticity is below two, there will be decreasing returns to scale in \((A, K, L)\) space, while for a demand elasticity greater than two, we can have any of the three cases of decreasing, constant or increasing returns to scale depending on the assumption made about \( \nu + \omega \). As it is usually assumed that the returns to scale in the production function are not too low, a moderate demand elasticity will usually imply increasing returns to scale in the revenue function in \((A, K, L)\) space.

\(^{16}\)Their profitability shocks combine demand, TFP and factor price shocks. However, their assumptions about the exponents of the profit function imply \( c = 1 - b \) within the set-up of this paper.
2.2 The Static Labor Input Decision and Profits

Throughout this paper it is assumed that labor input can be freely adjusted within each period and becomes immediately available for production.\textsuperscript{17} In contrast to that, new capital is assumed to take one period to be ready for use in production, so that the capital stock is taken as fixed in the current period by the firm. With these assumptions, the labor input decision of the firm becomes static and is determined by the maximization of gross profits given the capital stock and the shock to revenues. Taking the first-order condition and solving for $L_{i,t}$ gives us the following function for the optimal labor input decision:

$$L(w_t, A_{i,t}, K_{i,t}) = \kappa_t A_{i,t}^{\frac{c}{1+b}} K_{i,t}^{\frac{n}{1+b}}$$  \hspace{1cm} (2)

where $\kappa_t = b^{\frac{1}{1+b}}w_t^{-\frac{1}{1+b}}$ is a parameter that depends on the wage. Moreover, plugging the labor policy function into the revenue function and into the equation for gross profits yields the following reduced form revenue and profit functions:

$$R(w_t, A_{i,t}, K_{i,t}) = \chi_t A_{i,t}^{\frac{c}{1+b}} K_{i,t}^{\frac{n}{1+b}}$$  \hspace{1cm} (3)

$$\Pi(w_t, A_{i,t}, K_{i,t}) = \phi_t A_{i,t}^{\frac{c}{1+b}} K_{i,t}^{\frac{n}{1+b}}$$  \hspace{1cm} (4)

where $\chi_t = b^{\frac{b}{1+b}}w_t^{-\frac{b}{1+b}}$ and $\phi_t = (1-b)b^{\frac{b}{1+b}}w_t^{-\frac{b}{1+b}}$ are again parameters that depend on the wage. Note that equations (2), (3) and (4) all have the same functional form in $(A, K)$ space. Due to the assumptions of either decreasing returns to scale in production or some degree of market power, which both imply that $0 < a + b < 1$, all three functions will be concave in $K$. Concavity or convexity in $A$ and whether we have increasing, constant or decreasing returns to scale in $(A, K)$ space will however depend crucially on the value of $c$, i.e. on the choice of whether we consider demand or supply shocks.

Relating to the discussion above, the assumption of $c = 1 - a - b$ in Bloom (2009) implies concavity in $A$ and constant returns to scale in $(A, K)$. In contrast, if we set $c = 1$ as in Bloom et al. (2010) and Bachmann and Bayer (2011), the functions will be convex in $A$ and have increasing returns to scale in $(A, K)$. Finally, for $c = 1 - b$ as in Cooper and Haltiwanger (2006) and Caballero and Engel (1999), the functions will be linear in $A$ and have increasing returns to scale in $(A, K)$ space. How these different assumptions determine the effect of uncertainty shocks is shown analytically in section 3 for the case without capital adjustment costs and numerically in section 4 for the case with capital adjustment costs.

For the rest of the paper it is assumed that the wage rate does not vary over time. This is mainly done to keep the model as tractable as possible. Variations in wages could however be easily implemented via making them a function of the aggregate profitability shock and/or the uncertainty shock. With the assumption of a constant wage the variables $\kappa_t$, $\chi_t$ and $\phi_t$ become constants, which allows us to drop the wage rate from the state space of the firm. As the shock to revenues is

\textsuperscript{17}This is a common assumption in most of the investment literature. See for example Hayashi (1982), Abel and Eberly (1994), Bertola and Caballero (1994), Abel and Eberly (1996), Caballero and Engel (1999), Cooper and Haltiwanger (2006) or Bloom et al. (2007) among others.
now the only stochastic variable left in the model and it directly affects profits, it will from now on be labeled as a profitability shock.

### 2.3 The Stochastic Process for Profitability

As is standard in the investment literature with heterogeneous firms, the profitability of each firm \( A_{i,t} \) is assumed to be the product of an aggregate \( Z_t \) and an idiosyncratic \( \Psi_{i,t} \) component.\(^{18}\) Furthermore, both the aggregate and idiosyncratic components are assumed to follow persistent AR(1) processes in logs, which is consistent with US micro data as shown by Cooper and Haltiwanger (2006). Finally, in line with the models by Bloom (2009), Bloom et al. (2010), Bachmann and Bayer (2011), Arellano et al. (2011), Gilchrist et al. (2009) and Vavra (2012) uncertainty shocks are incorporated into the model through changes in the variance of idiosyncratic shocks.\(^{19}\) Given these assumptions the profitability process can be described by the following equations:

\[
A_{i,t} = Z_t \Psi_{i,t}
\]

\[
z_t = \mu_z + \rho_z z_{t-1} + \eta_t
\]

\[
\psi_{i,t} = \mu_\psi + \rho_\psi \psi_{i,t-1} + \upsilon_{i,t}
\]

Here, a lower case letter refers to the logarithm of the variable and \( \eta_t \sim \mathcal{N}(\mu_\eta, \sigma^2_\eta) \) and \( \upsilon_{i,t} \sim \mathcal{N}(\mu_{\upsilon}, \sigma^2_{\upsilon}) \) are i.i.d. innovations to aggregate and idiosyncratic profitability. Note that the idiosyncratic profitability process is specified as a markov-switching process, where it is assumed that the standard deviation of idiosyncratic shocks follows a two-point markov chain with support \( \sigma_{\upsilon,t} \in \{\sigma^L_{\upsilon}, \sigma^H_{\upsilon}\} \) and transition probabilities \( \Pr(\sigma_{\upsilon,t+1} = \sigma^j_{\upsilon} | \sigma_{\upsilon,t} = \sigma^i_{\upsilon}) = \pi_{ij} \) between the high and low uncertainty state. Embedded in the above specification is the standard timing assumption that the variance of idiosyncratic shocks is known one period in advance with certainty.\(^{20}\) This implies that agents always know the true variance of shocks applicable in the next period and hence all the variations in uncertainty perceived by firms are rational in the sense that they are related to more volatility in the shocks to fundamentals. Finally, this standard timing assumption for information implies that the expectational effect of uncertainty shocks leads the distributional effect by one period. I.e. firms’ expectations about the future get more uncertain on impact of the uncertainty shock, but the distributional effect only starts with a one period delay once firms start drawing shocks from a more dispersed distribution. Note also that the mean of idiosyncratic profitability shocks is allowed to vary along side the variance of shocks. As is shown in Appendix C this is done in order to correct for the positive effect on mean expectations that an increase in the shock variance causes given that the process is specified in logs.

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\(^{18}\)See table 1 for a summary of common modeling assumptions in the investment literature.

\(^{19}\)In principle, we could also assume that the variance of aggregate shocks changes over time. However, as Bloom et al. (2010) argue, this would mainly affect conditional heteroskedasticity of aggregate variables. As the main motivation of the Uncertainty Shocks literature is that cross-sectional measures of spread at the micro level vary over time, it seems sufficient to incorporate firm level uncertainty shocks. Hence, to simplify the model, the variance of shocks to the aggregate component is assumed to be constant.

\(^{20}\)See all of the papers on uncertainty shocks mentioned above.
2.4 Specification of Capital Adjustment Costs

The firm’s capital stock is fixed within each period, as it is assumed to take one period for new capital to be installed and ready for production. Moreover, capital is assumed to depreciate at the rate $\delta$ per period, so that the law of motion for the capital stock is given by the following equation, where $I$ denotes the level of investment:

$$K_{i,t+1} = K_{i,t}(1 - \delta) + I_{i,t}$$  \hspace{1cm} (8)

In line with the papers by Cooper and Haltiwanger (2006) and Bloom (2009) it is assumed that the firm faces convex and non-convex costs of adjusting the capital stock, as well as partial investment irreversibilities. Both, adjustment costs and partial irreversibilities bring interesting real options effects to the capital accumulation process and in particular will determine how the expectational effect of uncertainty shocks impacts on investment decisions. The adjustment cost function and the price of capital can be represented by the following equations:

$$C(A_{i,t}, K_{i,t}, I_{i,t}) = \gamma_2 \left( \frac{I_{i,t}}{K_{i,t}} \right)^2 K_{i,t} + (1 - \lambda) \Pi(A_{i,t}, K_{i,t})1_{\{I_{i,t} \neq 0\}} + FK_{i,t}1_{\{I_{i,t} \neq 0\}}$$  \hspace{1cm} (9)

$$p(I_{i,t}) = \begin{cases} p_s, & \text{if } I_{i,t} < 0 \\ p_b, & \text{if } I_{i,t} > 0 \end{cases}$$  \hspace{1cm} (10)

The convex adjustment costs are assumed to be quadratic while the non-convex adjustment costs can be a fraction of current profits $(1 - \lambda)$ or a fraction of the capital stock $(F)$.

Finally, $p_s$ denotes the selling price of capital and $p_b$ the buying price and it is assumed that $p_s < p_b$ so that there are partial irreversibilities for investment.

2.5 The Bellman Equation of the Firm

As mentioned above, firms are assumed to be risk neutral in order to isolate the effects of uncertainty shocks that are due to adjustment frictions. Moreover, it is assumed that the discount rate, i.e. the interest rate, with which the firms discount their expected future profit streams is constant over time. In reality of course, the interest rate varies over time and should in particular be related to the business cycle. Incorporating such interest rate changes are again easily implemented as functions of the aggregate and uncertainty shock. However, to isolate the pure effects of uncertainty shocks it was decided to abstract from factor price changes.

Given the objects defined in the previous subsections, the dynamic decision problem for each firm of maximizing the present discounted value of expected profits can be summarized by the following Bellman Equation. To save on notation the firm subscript $i$ for each variable is omitted and primes denote next period variables:

$$V(A, K, \sigma_\nu) = \max_I \Pi(A, K) - C(A, K, I) - p(I)I + \beta E_{\nu'} \Pi[A', \sigma_{\nu'} | A, \sigma_\nu] [V(A', K', \sigma_{\nu'})]$$  \hspace{1cm} (11)

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21 Both papers find that all these forms of capital adjustment frictions are needed to match the micro data on investment behavior by plants/firms.

22 $1_{\{I_{i,t} \neq 0\}}$ is an indicator function that takes a value of 1 whenever investment is nonzero, and a value of 0 otherwise.

23 In the Bellman Equation below total profitability is used as the state variable to save on notation. It should be kept in mind however that in order to solve the model, information on both
Here, $\beta$ is the period discount factor, $\Pi(A,K)$ is the reduced form profit function, $C(A,K,I)$ captures investment adjustment costs, $p(I)$ is the effective price of newly installed or retired capital, and next period capital is given by the law of motion in equation (8). The solution to this Bellman Equation will yield a policy function for investment or alternatively next period capital of the form $I(A,K,\sigma_v)$ and $K'(A,K,\sigma_v)$.

2.6 Dynamic Aggregation of the Firm Distribution

The Bellman Equation (11) implies that the relevant state vector for each firm consists of aggregate profitability, idiosyncratic profitability, the capital stock and the uncertainty shock. In order to determine the relevant state vector that determines the behavior of aggregate variables at each point in time, first note that firms in our model differ in two respects from each other, namely idiosyncratic profitability and capital. The aggregates in our economy will therefore be characterized by a joint distribution of the form $G_t(\Psi_{i,t},K_{i,t})$ in every period. Let the associated density be denoted by $g_t$. The density $g_t$ is indexed by $t$ to make it explicit that it will change over time due to uncertainty shocks. In addition to this density, the aggregate profitability shock and the current variance regime will determine how aggregate variables behave. Following this logic, the aggregate of a generic endogenous variable $X$ can be expressed as a function of the following state vector $S_t = [g_t(\Psi_{i,t},K_{i,t}),Z_t,\sigma_{v,t}]$:

$$E[X_{i,t}|S_t] = \bar{X}_t(S_t) = \int g_t(\Psi_{i,t},K_{i,t})X_{i,t}(\Psi_{i,t},K_{i,t},Z_t,\sigma_{v,t})d\Psi dK$$ (12)

Moreover, with this definition of the aggregate state vector we can concisely represent the evolution of the density $g_t$ over time. First note that $g_t$ will depend on the joint density of idiosyncratic profitability and capital at time $t-1$. Moreover, as the capital in the next period is a function of aggregate profitability and the uncertainty shock, $Z_{t-1}$ and $\sigma_{v,t-1}$ will also affect $g_t$. Finally, the distribution of idiosyncratic profitability in the current period depends on last period’s distribution and the uncertainty shock. But this simply amounts to saying that the joint density of idiosyncratic profitability and capital at time $t$ depends on the aggregate state vector at time $t-1$. We can therefore express $g_t$ as a conditional distribution of $S_{t-1}$:

$$g_t(\Psi_{i,t},K_{i,t}) = g(\Psi_{i,t},K_{i,t}|S_{t-1})$$ (13)

2.7 Formal Definition of Expectational and Distributional Effects

In general, there are two different channels through which uncertainty shocks can affect aggregate investment and other endogenous variables in the model. On the one hand, the level of uncertainty can influence the investment behavior of each
individual firm through it’s effect on the expected dispersion of future profitability. For example, in the presence of fixed costs higher uncertainty will lead to more inaction due to real options effects. On the other hand, changes in the level of uncertainty can affect aggregate investment through their effect on the dispersion of idiosyncratic profitability across firms. For example, given the timing assumption implicit in equation (7) a positive uncertainty shock will increase the cross-sectional dispersion of idiosyncratic profitability in the subsequent period. This change in the dispersion of profitability across firms can in turn affect aggregate investment whenever investment policy functions are not linear in idiosyncratic profitability.

Similar to the terminology used by Bloom (2009), the former channel will be referred to as the expectational effect, while the latter channel will be referred to as the distributional effect of uncertainty shocks. \(^{24}\) Hence, the expectational effect will be related to changes in the standard deviation of the forecast distributions of firms, while the distributional effect will be related to changes in the cross-sectional dispersion of idiosyncratic profitability. The overall influence of an uncertainty shock on endogenous aggregate variables will therefore be determined by the sum of both effects. Given the timing assumption that firms know the realization of the variance regime one period in advance, the expectational effect will lead the distributional effect by one period. I.e. when a positive uncertainty shock occurs, firms’ expectations change upon impact, but the cross-sectional dispersion of idiosyncratic profitability does not change until the next period, when firms start drawing shocks from a more dispersed distribution.

In order to formally define expectational and distributional effects, it is useful to rewrite the joint density \(g_t\) as the product of a marginal and a conditional density:

\[
g_t(\Psi_{i,t}, K_{i,t}|S_{t-1}) = f(\Psi_{i,t}|S_{t-1}) \cdot h(K_{i,t}|\Psi_{i,t}, S_{t-1})
\]

Using this in equation (12) we can rewrite the expression for a generic aggregate variable as follows:

\[
\bar{X}_t = \int \int f(\Psi_{i,t}|f_{t-1}, \sigma_v,t-1) \cdot h(K_{i,t}|\Psi_{i,t}, g_{t-1}, Z_{t-1}, \sigma_v,t-1) \cdot X_{i,t}(\Psi_{i,t}, K_{i,t}, Z_{t}, \sigma_v,t)d\Psi dK
\]

(14)

Note that use has been made of the fact that the distribution of idiosyncratic profitability at time \(t\) will only depend on last period’s distribution and variance regime and not on the whole state vector \(S_{t-1}\). Taking this formulation for aggregate variables as a basis, the dynamics due to the expectational and distributional effect can be respectively defined as follows:

\[
\bar{X}_t^E = \int \int f(\Psi_{i,t}|f_{t-1}, \sigma_v^L) \cdot h(K_{i,t}|\Psi_{i,t}, g_{t-1}, Z_{t-1}, \sigma_v^L) \cdot X_{i,t}(\Psi_{i,t}, K_{i,t}, Z_{t}, \sigma_v^L)d\Psi dK
\]

(15)

\[
\bar{X}_t^D = \int \int f(\Psi_{i,t}|f_{t-1}, \sigma_v,t-1) \cdot h(K_{i,t}|\Psi_{i,t}, g_{t-1}, Z_{t-1}, \sigma_v) \cdot X_{i,t}(\Psi_{i,t}, K_{i,t}, Z_{t}, \sigma_v^L)d\Psi dK
\]

(16)

\(^{24}\)The respective labels used by Bloom (2009) are uncertainty and volatility effect.
3 Analytical Results in the Absence of Adjustment Costs

In order to get some intuition on how the model described in the previous section behaves, it is useful to first consider the case without adjustment costs and irreversibilities. For that case it is possible to derive analytic expressions for the capital policy function and the dynamics of aggregate variables. The insights gained from this exercise can then be used as a benchmark to which we can compare the simulation results of models with capital adjustment frictions. Particular focus is put on the analysis of the expectational and distributional effects of uncertainty shocks and how they depend on whether we consider them to demand or TFP.

3.1 The Capital Policy Function

In the absence of capital adjustment costs and irreversibilities, the firm’s decision problem allows for an analytical solution. Taking the first-order condition of the Bellman Equation in (11) and combining it with the envelope condition yields the following optimal capital policy function:

\[
K'(A, \sigma_v) = \varphi E \left[ A^{1-c-\frac{1-b}{1-a-b}} | A, \sigma_v \right]
\]

where \( \varphi = [(a\beta\phi)/(1 - b)(p - p\beta(1 - \delta))]^{1+b} \). From this expression we can already see that even without adjustment costs there can be expectational and distributional effects of uncertainty shocks on capital accumulation. For instance, whenever \( c \) is greater (smaller) than \( 1 - b \), the resulting convexity (concavity) of the profit function in \( A \) implies that more dispersion in future profitability shocks will increase (decrease) the desired capital stock of a single firm. Hence, the expectational effect of uncertainty shocks will be positive whenever \( c > 1 - b \), which is a direct implication of Jensen’s inequality. The curvature of the profit function with respect to profitability is in turn determined by the returns to scale of the revenue function (1) in \((A, L)\) space. To be precise, whenever \( c \) is greater (smaller) than \( 1 - b \) there are increasing (decreasing) returns to scale in \((A, L)\) space, the profit function is therefore convex (concave) in \( A \) and hence the expectational effect is positive (negative). Note that this result holds even though it was assumed that there are no adjustment costs.

This potentially positive effect of higher uncertainty has been first shown by Hartman (1972) and Abel (1983), who found that under the assumptions of constant returns to scale, perfect competition and an increasing adjustment cost function, more uncertainty in the price leads to higher investment due to the convexity of the profit function in the price.\(^{26}\) In order to determine the distributional effect of

\(^{25}\)A detailed derivation of this policy function and subsequent results can be found in Appendix D.

\(^{26}\)In their framework adjustment costs are needed to make the size of the firm determinate, which is not needed in our case due to the assumption of decreasing returns to scale. The case of \( c = 1 \) in the model above is qualitatively the same as their model.
uncertainty shocks without adjustment costs for the current model it is useful to derive an analytic expression for the expectation in equation (17):^{27}

\[ K'(Z, \Psi, \sigma) = \varphi \left( e^{\mu \psi} e^{\mu \eta} e^{\frac{a^2}{2} + \sigma^2} (Z \rho \psi)^{\frac{a}{a-1}} e^{\frac{a^2 c^2 (1-b)}{(1-b)(1-a-b)}} \right) (18) \]

With this capital policy in hand it is instructive to analyze how the assumptions about the parameter \( c \) determine the sign and magnitude of the distributional effect of uncertainty shocks. As can be easily seen from the policy function, the distributional effect is positive (negative) whenever \( c \) is greater (smaller) than \( 1 - a - b \), which is due to the resulting convexity (concavity) of the policy function in idiosyncratic profitability and therefore more cross-sectional dispersion in profitability will increase the aggregate capital stock.\(^{28}\) This in turn simply means that the distributional effect is positive (negative) whenever the revenue function (1) displays increasing (decreasing) returns to scale in \((A, K, L)\) space. The intuition behind this result is that whenever there are increasing RTS in the revenue function, ex-post higher cross-sectional dispersion in the fixed factor, i.e. profitability, is a time of opportunity for firms to invest. Firms that receive large negative shocks reduce investment proportionately less than by how much firms that receive large positive shocks increase investment.

### 3.2 Expectational and Distributional Effects for Demand and TFP

With the previous general considerations in mind, we can analyze expectational and distributional effects of uncertainty shocks for the three cases of \( c \in \{1, 1-b, 1-a-b\} \) commonly used in the literature. First, recall that increasing RTS in the revenue function are only possible for TFP shocks and not for demand shocks.\(^{29}\) For example, when we consider a calibration where the production function has constant RTS with exponents of 1/3 and 2/3 on capital and labor, and the demand elasticity is set equal to four, then \( a = 0.25, b = 0.5, \) and \( c = 0.25 = 1 - a - b \), i.e. there are constant RTS for demand shocks and \( c = 0.75 = 1 - a \), i.e. there are increasing RTS for supply shocks. In fact the specification by Bloom (2009) is exactly the one just described for demand shocks. For this case the desired capital stock is now decreasing in the variance of shocks, so there is a negative expectational effect. Moreover, the distributional effect is slightly negative because the desired capital stock is concave

---

\(^{27}\)For the derivation it is assumed that the mean of idiosyncratic innovations changes at the same time as the variance. In particular it is assumed that \( \mu_{\psi,t-1} = -\sigma^2_{\psi,t-1}/2 \), which ensures that the one period ahead expected value is not affected by the increase in the innovation variance. This adjustment will be called a Jensen correction in the rest of the paper, as the increase in the expected mean due to a higher variance is due to Jensen’s inequality. See Appendix C for details.

\(^{28}\)With the stochastic process for profitability assumed in this paper, the persistence parameter \( \rho \psi \) also affects the sign of the volatility effect. As it is common to assume highly persistent or even random walk processes for idiosyncratic profitability, the effect of the persistence parameter is ignored in this discussion.

\(^{29}\)Recall from section 2.1 that for constant RTS in the production function and a low elasticity of demand of only two, the revenue function for TFP shocks already has constant RTS and the RTS are increasing in the value of the demand elasticity.
in profitability. This is due to the fact that \( \frac{c}{1-a-b} = 1 \) and \( \rho \psi \) is smaller than one. Hence, for a random walk process, the distributional effect would be zero.

If we have instead that \( c = 1-b \), as implicitly assumed in Cooper and Haltiwanger (2006), then a higher variance of shocks has no effect on the desired capital stock of each firm holding everything else equal.\(^{30}\) In other words, the expectational effect is zero. However, as long as the AR(1) persistence parameter is sufficiently high there is a positive distributional effect. This can be seen by noting that the desired capital stock is convex in idiosyncratic profitability because \( \frac{c}{1-a-b} > 1 \). This convexity implies that more dispersion in idiosyncratic profitability across firms increases the aggregate capital stock.

Now consider the case of \( c = 1 \), as in Bachmann and Bayer (2011) and Bloom et al. (2010), which corresponds to perfect competition, decreasing RTS in the production function and TFP shocks. In this case, a larger variance of shocks increases the desired capital stock of each firm, i.e. the expectational effect is positive. In addition, the desired capital stock is convex in profitability, so the distributional effect is also positive. The implications of these three cases of \( c \) for expectational and distributional effects of uncertainty shocks are summarized in table 2.

### Table 2: Distributional and expectational effects without adjustment costs

<table>
<thead>
<tr>
<th></th>
<th>( c = 1 - a - b )</th>
<th>( c = 1 - b )</th>
<th>( c = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distributional Effect</strong></td>
<td>( \frac{c}{1-a-b} = 1 )</td>
<td>( \frac{c}{1-a-b} &gt; 1 )</td>
<td>( \frac{c}{1-a-b} &gt; 1 )</td>
</tr>
<tr>
<td><strong>Expectational Effect</strong></td>
<td>( \frac{c^2-c(1-b)}{(1-b)(1-a-b)} &lt; 0 )</td>
<td>( \frac{c^2-c(1-b)}{(1-b)(1-a-b)} = 0 )</td>
<td>( \frac{c^2-c(1-b)}{(1-b)(1-a-b)} &gt; 0 )</td>
</tr>
</tbody>
</table>

It has therefore been shown that even in the absence of capital adjustment costs, aggregate investment can be influenced in different ways by variations in uncertainty. We can broadly distinguish between an expectational and a distributional effect. The former has to do with the fact that an uncertainty shock affects the expected dispersion in future profitability, while the latter has to do with the fact that uncertainty shocks affect the dispersion in actual profitability with a one period delay. For the Cooper and Haltiwanger (2006) and the Bachmann and Bayer (2011) specifications of \( c \) which can be both interpreted as TFP shocks, higher uncertainty should lead to more investment because of a positive distributional effect and a non-negative expectational effect, while in the Bloom (2009) specification the uncertainty shocks to demand will reduce aggregate investment as the expectational effect is negative and the distributional effect is zero.

### 3.3 The Dynamics of the Profitability Distribution

Before moving to the simulation of various models, it is useful to better understand the dynamics of the profitability distribution that are induced by uncertainty shocks.

\(^{30}\)Cooper and Haltiwanger (2006) use a profit function with an exponent of one on profitability which corresponds to \( c = 1 - b \) in my framework. Hence, there are increasing RTS in the revenue function and shocks are therefore interpreted as TFP shocks, although in an environment with some degree of market power.
It has been mentioned above that the mean of idiosyncratic profitability shocks needs to be adjusted at the same time as the variance, in order for uncertainty shocks to correspond to mean preserving spreads.\(^{31}\) As will be shown below, it is however not easy to avoid any type of mean effects of changes in the shock variance for a log-normal process. Given the AR(1) process in logs assumed in equation (7), the distribution of idiosyncratic profitability across firms will be log-normal. It can easily be shown that the dynamics of the mean and variance of log idiosyncratic profitability are governed by the following equations:\(^{32}\)

\[
E[\psi_{i,t}] = \frac{\mu_\psi + \mu_L^L}{1 - \rho_\psi} + \sum_{j=1}^{\infty} \rho_\psi^{j-1} \Delta \mu_{v,t-j}
\]

\[
V[\psi_{i,t}] = \frac{(\sigma_L^L)^2}{1 - \rho_\psi^2} + \sum_{j=1}^{\infty} \rho_\psi^{2(j-1)} \Delta \sigma_{v,t-j}^2
\]

Here, the variable \(\Delta \mu_{v,t} = \mu_{v,t} - \mu_L^L\) captures the difference between the mean of idiosyncratic shocks under the current regime and the low regime. Analogous to this the variable \(\Delta \sigma_{v,t}^2 = \sigma_{v,t}^2 - (\sigma_L^L)^2\) captures the difference between the variance of idiosyncratic shocks under the current regime and the low regime. It is obvious from the equations that the mean and variance of log idiosyncratic profitability are given by the sum of the value that would prevail under the low uncertainty regime and the accumulated effect due to occasional switches to the high uncertainty regime. In the end we are however interested in the evolution of the distribution of idiosyncratic profitability in levels and not in logs. Given that log idiosyncratic profitability is normally distributed, it is easy to establish a mapping from the moments in logs to the moments in levels:

\[
E[\Psi_{i,t}] = e^{E[\psi_{i,t}]+V[\psi_{i,t}]/2}
\]

\[
V[\Psi_{i,t}] = e^{2E[\psi_{i,t}]+2V[\psi_{i,t}] - e^{2E[\psi_{i,t}]+V[\psi_{i,t}]}}
\]

If we now consider the case where the mean of shocks does not change over time, it is easy to see that the mean and variance of idiosyncratic profitability in levels will go up after an uncertainty shock. For this case an uncertainty shock also leads to an increase in the expected value of profitability for each firm, as is shown in Appendix C. In order to keep the one-period-ahead expected value of profitability unaffected by switches in the variance we can set \(\mu_{v,t} = -\sigma_{v,t}^2/2\), which is equivalent to saying that \(\Delta \mu_{v,t} = -\Delta \sigma_{v,t}^2/2\). But even with this so-called Jensen correction, the cross-sectional mean of idiosyncratic profitability will not stay constant over time whenever \(0 < \rho_\psi < 1\).

To understand this, note that the mean of idiosyncratic profitability only stays constant if the log mean changes in exactly an off-setting way to the log variance in all periods. But from equations (19) and (20) we can immediately see that whenever \(0 < \rho_\psi < 1\), the two moments do not change in offsetting ways in all periods after

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\(^{31}\)In other words we are interested in pure second moment shocks without any first moment implications. The fact that the expected value of a log-normally distributed variable increases in the variance of shocks is shown in Appendix C.

\(^{32}\)See Appendix E for a detailed derivation of these moments.
an uncertainty shock. To be precise, in the first period after an uncertainty shock
the increase in the log variance is exactly offset by the fall in the log mean. In
subsequent periods however, the log variance reverts quicker to the initial level than
the log mean due to the presence of $\rho_\psi^2$. Therefore, the cross-sectional mean in
levels decreases compared to the low uncertainty value after a positive uncertainty
shock. The dynamics of the cross-sectional mean and standard deviation for the two
cases just described are illustrated in the top left panels of figures 1 and 2 for an
uncertainty shock that hits in period zero and lasts for five periods. This shows that
unless we work with a random-walk, it is hard to model mean-preserving spreads
for driving processes in logs.

3.4 Simulation Results for various Parameterizations

The three different models with $c \in \{1, 1 - b, 1 - a - b\}$ are simulated in this section
in order to illustrate quantitatively how the different assumptions about the RTS
of the revenue function and hence about the nature of the underlying structural
shocks affect aggregate investment, employment and revenues. The case of $c = 1$
used by Bachmann and Bayer (2011) is labeled as a pure TFP shock, while the
case of $c = 1 - b$ used by Cooper and Haltiwanger (2006) is labeled as a TFP
shock with market power, and the case of $c = 1 - a - b$ used by Bloom (2009) is
labeled as a demand shock. Moreover, because it matters whether we apply a Jensen
correction or not to the idiosyncratic profitability process, both cases are examined.
The parameter values used for the simulation exercise are summarized in table 3
and discussed further below in section 4.1.

To study the effect of an uncertainty shock quantitatively the following simula-
tion exercise is performed. Aggregate shocks are turned off and uncertainty is set
to the low regime for the infinite past. In period zero an uncertainty shock hits
the system that almost doubles the idiosyncratic shock variance. This uncertainty
shock lasts for five consecutive periods and uncertainty is low again for all subse-
quent periods.\textsuperscript{33} Figure 1 compares the responses of aggregate variables for the case
without a Jensen correction, while figure 2 displays the responses for the case with
a Jensen correction.

It is obvious from the figures that the higher the value of $c$, the higher the posi-
tive response of the variables to the uncertainty shock. This has to do with the fact
that both the expectational and the distributional effects of uncertainty shocks are
increasing in $c$, i.e. in the RTS of the revenue function. Furthermore, it can be seen
that uncertainty shocks to TFP lead to considerable booms in investment, employ-
ment and revenues, while uncertainty shocks to demand lead to falls in aggregates
only for the case where a Jensen correction is applied to the driving process. Quanti-
tatively, the burst in aggregate investment after an uncertainty shocks to pure TFP
is around 250 % to 400 %, while it is 50 % to 100 % for uncertainty shocks to TFP
with market power, and $\pm 10 \%$ for shocks to demand.

In summary, we can say that higher uncertainty is a rather positive phenomenon
when the uncertainty is to TFP, while uncertainty shocks to demand can have neg-

\textsuperscript{33}The probability of an uncertainty shock lasting for five consecutive periods is slightly above
50 % for the parameterization used.
Figure 1: The impact of uncertainty shocks in the model without ACs (no Jensen correction)

Figure 2: The impact of uncertainty shocks in the model without ACs (with Jensen correction)
ative aggregate implications. The positive aggregate effects of uncertainty shocks to TFP are not only due to the convexity of profits in TFP which causes a positive expectational effect as pointed out by Hartman (1972) and Abel (1983). In addition, increasing returns to scale in \((A,K,L)\) space for TFP shocks induce a positive distributional effect. For demand shocks, the RTS of the revenue function are always lower or equal to one and hence the distributional effect will never be positive for demand shocks when there are no adjustment costs present.

Now that it is understood how uncertainty shocks to demand and TFP affect aggregate variables in the model without adjustment costs we can move on to consider how the dynamics of aggregate variables in response to uncertainty shocks are affected by the presence of capital adjustment frictions. Due to real-options effects in the presence of non-convex adjustment costs, the expectational effect of uncertainty shocks will be altered considerably, while the distributional effect will also be somewhat affected by the presence of a discrete choice for investment.

4 Simulation Results for the General Model

Because it is no longer possible to derive closed form expressions for the capital policy function in the presence of capital adjustment costs, numerical methods are employed for the analysis.\(^{34}\) For this purpose the same three versions of \(c\) as above are solved for three different adjustment cost parameterizations that have been estimated in the literature. The next subsection provides a discussion of the parameter values that are used to calibrate the model. This is followed by an analysis of how adjustment costs change the investment policy functions of firms in the presence of time-varying uncertainty. Finally, the aggregate dynamics induced by uncertainty shocks are analyzed. Particular focus is again put on disentangling expectational and distributional effects of uncertainty shocks.

4.1 Discussion of Parameters

There are two dimensions in which the parameters are changed between the different models that are solved and simulated: adjustment costs and the RTS of the revenue function in \((A,K,L)\) space via \(c\). A summary of all the other parameters that are not varied across the simulations can be found in table 3. Virtually all of these parameter values are taken from the paper by Bloom et al. (2010), who calibrate a general equilibrium business cycle model with capital adjustment costs, labor adjustment costs and uncertainty shocks. The reason for this choice of parameters is to stay as close as possible to the existing uncertainty shocks literature. In the remainder of the paper all models will feature a Jensen correction in the mean of idiosyncratic shocks in order to allow for the biggest possible negative effect of uncertainty shocks.

The model frequency is one quarter and the depreciation rate is set to 2.6 % per period. Regarding the revenue function, it is assumed that there are decreasing returns to scale in \((K,L)\) space, with the exponent on capital set to 0.25 and the

\(^{34}\)See Appendix F for more information on the algorithm and the accuracy of the approximation.
exponent on labor set to 0.5.\textsuperscript{35} Both aggregate and idiosyncratic profitability processes are assumed to be highly persistent with AR(1) coefficients of around 0.96. The low standard deviation of idiosyncratic profitability shocks is set to 6.7\% and this standard deviation almost doubles in the high uncertainty state. Moreover, both uncertainty regimes are fairly persistent, with the probability of staying in the low uncertainty state being 95\% and the probability of staying in the high uncertainty state being 88.5\%. There are two parameters that are set slightly different from Bloom et al. (2010): The standard deviation of aggregate shocks is set to 1.5\% and the discount rate is set to 0.99.\textsuperscript{36} Finally, two arbitrary normalizations are performed through setting the wage rate and the buying price of capital equal to one.

Table 3: Common parameters across the simulations

<table>
<thead>
<tr>
<th>Par</th>
<th>Value</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>0.99</td>
<td>Discount factor</td>
<td>Own choice</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.026</td>
<td>Depreciation rate</td>
<td>Bloom et al. (2010)</td>
</tr>
<tr>
<td>(w)</td>
<td>1</td>
<td>Wage rate</td>
<td>Normalization</td>
</tr>
<tr>
<td>(a)</td>
<td>0.25</td>
<td>Exponent on capital</td>
<td>Bloom et al. (2010)</td>
</tr>
<tr>
<td>(b)</td>
<td>0.50</td>
<td>Exponent on labor</td>
<td>Bloom et al. (2010)</td>
</tr>
<tr>
<td>(p_{bh})</td>
<td>1</td>
<td>Buying price of capital</td>
<td>Normalization</td>
</tr>
<tr>
<td>(\mu_z)</td>
<td>0</td>
<td>Intercept of aggregate profit.</td>
<td>Bloom et al. (2010)</td>
</tr>
<tr>
<td>(\rho_z)</td>
<td>0.9627</td>
<td>AR(1) parameter of aggregate profit.</td>
<td>Bloom et al. (2010)</td>
</tr>
<tr>
<td>(\sigma_{\eta})</td>
<td>0.015</td>
<td>Std of innovations of aggregate profit.</td>
<td>Own choice</td>
</tr>
<tr>
<td>(\mu_{\psi})</td>
<td>0</td>
<td>Intercept of idiosyncratic profit.</td>
<td>Bloom et al. (2010)</td>
</tr>
<tr>
<td>(\rho_{\psi})</td>
<td>0.9627</td>
<td>AR(1) parameter of idiosyncratic profit.</td>
<td>Bloom et al. (2010)</td>
</tr>
<tr>
<td>(\sigma_{L_v})</td>
<td>0.0671</td>
<td>Low Std of innovations of idio. profit.</td>
<td>Bloom et al. (2010)</td>
</tr>
<tr>
<td>(\sigma_{H_v})</td>
<td>1.93 \cdot \sigma_{L_v}</td>
<td>High Std of innovations of idio. profit.</td>
<td>Bloom et al. (2010)</td>
</tr>
<tr>
<td>(\pi_{LL})</td>
<td>0.953</td>
<td>Probability of staying in low Std regime</td>
<td>Bloom et al. (2010)</td>
</tr>
<tr>
<td>(\pi_{HH})</td>
<td>0.885</td>
<td>Probability of staying in high Std regime</td>
<td>Bloom et al. (2010)</td>
</tr>
</tbody>
</table>

The three different adjustment cost specifications that are simulated are taken from the papers by Cooper and Haltiwanger (2006) and Bloom (2009) as these papers jointly estimate the rich set of capital adjustment costs and irreversibilities that is featured in the model at hand. The estimated adjustment cost specification of Cooper and Haltiwanger (2006), which is labeled ”C&H” in table 4, features considerable fixed costs of 20\% of profits, moderate resale losses of 1.9\% and

\textsuperscript{35}As mentioned above these exponents corresponds to a constant RTS production function with exponents of 1/3 and 2/3, and a demand elasticity of four. Moreover, this implies a curvature of the profit function in capital of 0.5, which is similar to the value estimated by Cooper and Haltiwanger (2006).

\textsuperscript{36}In the paper by Bloom et al. (2010) the aggregate profitability process is also subject to uncertainty shocks with a low standard deviation of 0.81\% and a high standard deviation of 3.49\%, while the discount rate is set to 0.996. As the high uncertainty state is less likely to occur than the low one, a constant standard deviation of 1.5\% was chosen, while the chosen discount rate of 0.99 speeds up the convergence of the value function, while it should not materially affect the results.
small quadratic adjustment costs of 0.153. The adjustment cost specification labeled "Bloom" in table 4 refers to the estimates of Bloom (2009) from a model that features capital and labor adjustment costs, while the adjustment cost specification labeled "Bloom 2" refers to the estimates that he obtains from a model with only capital adjustment costs. Both adjustment cost specifications feature large capital resale losses of around 40 % and moderate fixed costs of 1.5 % and 1.1 % of profits. The main difference between these two adjustment cost specifications is that the former does not feature quadratic adjustment costs while the latter features moderate quadratic adjustment costs of 0.996. Finally, the three different values of the curvature parameter on profitability of $c \in \{1, 1 - b, 1 - a - b\}$ are chosen due to their prevalence in the uncertainty shocks and capital adjustment cost literature.

Table 4: Parameters that are varied across the simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Adjusted Costs</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_s$</td>
<td>0.981</td>
<td>Selling price of capital</td>
</tr>
<tr>
<td>$F$</td>
<td>0.000</td>
<td>Investment fixed costs</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.796</td>
<td>% of profits kept</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.153</td>
<td>Quadratic adjustment cost</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Curvature on Profitability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>Demand TFP m.p. TFP Description</td>
</tr>
<tr>
<td></td>
<td>$1 - a - b$ $1 - b$ 1 Exponent on profitability</td>
</tr>
</tbody>
</table>

4.2 Characterizing the Solution of the Model

In order to understand how capital adjustment frictions change the expectational and distributional effects of uncertainty shocks, it is useful to first study the investment policy functions for the models. To this end, figure 3 plots investment as a function of the current capital stock for a given level of profitability under the low and high uncertainty regimes. The main feature that stands out is that a higher level of uncertainty leads to a larger investment inaction region for all three adjustment cost specifications, independent of whether we look at uncertainty shocks to demand or to TFP. This increased investment inaction in the presence of higher uncertainty is due to real-options effects that result from the presence of fixed costs and/or partial irreversibilities, which was first stressed in the seminal contribution by Bernanke (1983) and is a central feature of the models by Bloom (2009), Bloom et al. (2010) and Bachmann and Bayer (2011). Hence, the expectational effect of uncertainty shocks will be negative for all the specifications of the parameter $c$ and the various adjustment costs specifications.

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37 A discussion of the adjustment cost estimates of Cooper and Haltiwanger (2006) and of Bloom (2009) can be found in Appendix G.
38 See table 1 for a selective overview of assumptions that are routinely made in these two literatures.
Figure 3: Investment policy functions under high and low uncertainty

Notes: The policy functions for the adjustment cost specification "Bloom 2" are not shown here as they are qualitatively similar to the ones of the "C&H" specification. The specific policy functions plotted are for an average sized aggregate profitability shock and an above average level of idiosyncratic profitability. The main feature that the investment inaction region is larger for the high uncertainty state carries over to other levels of profitability.
In order to better understand the mechanism driving the distributional effect of uncertainty shocks figure 4 plots optimal investment for the Bloom adjustment cost specification as a function of idiosyncratic profitability.\textsuperscript{39} The important aspect to pay attention to is that the value of $c$ affects the shape of the investment policy function outside of the inaction regions. In particular, in line with what was derived for the case without adjustment costs, the investment policies are convex for pure TFP shocks and TFP shocks with market power and slightly concave for demand shocks outside of the inaction region. This implies that outside of the inaction regions the distributional effect is positive for $c = 1$ and $c = 1 - b$ and slightly negative for $c = 1 - a - b$. However, another important aspect to note is that due to the inaction regions, investment policies are locally convex for all three cases of $c$. This local convexity implies that even for $c = 1 - a - b$ the volatility effect will be positive as emphasized by Bloom (2009). Finally, the lower right panel of figure 4 plots the density of idiosyncratic profitability just before an uncertainty shock and after five consecutive periods of high uncertainty, which highlights the increase in dispersion of profitability that causes the distributional effect of uncertainty shocks.

\textsuperscript{39}The main finding that the value of $c$ shapes the curvature of the policy function outside of the inaction region carries over to the C&H and Bloom 2 adjustment cost specifications.
4.3 The Effect of Uncertainty on Investment, Employment and Revenues

Now that the investment policy functions and the dynamics of idiosyncratic profitability have been analyzed, we can move on to study the impulse responses of aggregate variables to an uncertainty shock for the various models under consideration. In order to obtain the impulse responses, a similar simulation exercise as in section 3.4 is performed for each of the models: Aggregate shocks are shut off and the distribution of idiosyncratic profitability is set to the unconditional asymptotic distribution under the low variance regime. In period zero an uncertainty shock is simulated to hit the economy lasting for five consecutive periods. Such a sequence of five consecutive periods of high uncertainty has a probability of slightly more than 50% for the uncertainty process at hand. For all subsequent periods uncertainty is set to the low regime again. This simulation exercise is performed for a panel of 10,000 firms and repeated 100 times. The results are then averaged across the 100 simulations for each of the model specifications.

The associated impulse responses of aggregate investment and revenues for each of the model specifications can be found in figure 5. Because the dynamics of aggregate revenues depend on the joint dynamics of profitability and the capital stock it is useful to start with the analysis of how an uncertainty shock affects investment in the various models. The first aspect to note is that the impulse responses for the C&H and Bloom adjustment cost specifications are qualitatively very similar for all three values of \( c \). In particular, no matter whether we look at uncertainty shocks to demand or supply, there is a large negative expectational effect on investment, which is evident from the considerable investment drops in period zero. Moreover, for all cases under consideration there is a subsequent rebound and overshoot in investment from period one onwards, which is due to a positive distributional effect that more than compensates for the negative expectational effect. Finally, there is a large spike in aggregate investment in period five due to the fading out of the negative expectational effect and the continued presence of the positive distributional effect.

Even though the qualitative behavior of aggregate investment is similar for uncertainty shocks to demand and TFP, the rebound and overshoot is much larger in the case of pure TFP shocks compared to demand shocks. This shows that the distributional effect of uncertainty shocks is increasing in the value of \( c \), i.e. in the returns to scale of the revenue function, as was pointed out in the previous subsection. Finally, it is worth to look at the magnitudes of the investment changes that are induced by the uncertainty shock. The initial investment drops in period zero

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The first 120 periods of the simulation are dropped to avoid initialization issues with the distribution of capital across firms.

Recall from section 2.2 that profits, revenues and labor input all have the same functional form in \((A,K)\) space and only differ by a constant shift parameter. Therefore, the dynamics of these three variables expressed in percentage deviations from their initial value will be identical. Hence, only the dynamics of aggregate revenues are shown. Note that the dynamics reported here are for gross revenues, i.e. adjustment costs are not subtracted from revenues.

Remember that period zero is the first period of the uncertainty shock so that only the expectational effect is present but not yet the distributional effect.
are mostly between 50 and 75 %, while the investment overshoots in period five are in the range of 140 to 440 %. These investment changes are very large, showing that variations in uncertainty can be of importance in models with fixed costs and irreversibilities.

The investment dynamics for the Bloom 2 adjustment cost specification are somewhat different to the ones described in the previous paragraph, due to the presence of higher quadratic adjustment costs. These quadratic adjustment costs make large adjustments of the capital stock undesirable which is evident from the fact that the drops and overshoots of investment are much smaller than for the other two adjustment cost specifications. In period zero, aggregate investment drops by around 20 to 30 % which indicates a negative expectational effect of the uncertainty shock. The overshoot of investment in subsequent periods is in the range of 10 to 90 % and again increasing in the value of \( c \).

For the impulse responses of aggregate revenues it is more useful to group the analysis by the type of uncertainty shock rather than by the adjustment cost specification. Starting with the case of pure TFP shocks, it is straightforward to see from figure 5 that none of the adjustment cost models leads to a drop in aggregate revenues after an uncertainty shock. Aggregate revenues actually experience a considerable boom of around 12 to 24 % by the end of period six depending on the adjustment cost specification. In order to better understand what is driving this result it is useful to recall that the revenue function is given by

\[
R(A_{i,t}, K_{i,t}) = \chi A_{i,t}^{c/(1-b)} K_{i,t}^{-a/(1-b)}. 
\]

When we consider pure TFP shocks, i.e. when \( c = 1 \), more dispersion in \( A \) will increase aggregate revenues due to convexity. In period one this direct positive effect of more dispersion dominates or at least offsets the negative effect on aggregate revenues due to a lower capital stock that is induced through the investment drop in period zero. In subsequent periods this direct positive distributional effect increases in strength due to the widening dispersion in TFP as the uncertainty shock persists. In addition, the positive distributional effect causes aggregate capital to increase which reinforces the rise in aggregate revenues even further. Thus, even though uncertainty shocks cause a one period drop in aggregate investment for pure TFP shocks, they lead to considerable booms in aggregate revenues, profits and labor.\(^{43}\)

Moving on to the case of TFP shocks in the presence of market power, we can see that an uncertainty shock leads to an initial drop in aggregate revenues of 0.5 to 1 % depending on the presence of moderate quadratic adjustment costs or not. Revenues then rebound quickly and reach their initial level after around four quarters. The subsequent overshoot is much larger than the initial drop, reaching between 1.5 and 6 % from quarter six onwards, when uncertainty about the future is low again and dispersion in profitability is at its maximum. For the case at hand recall that \( c = 1 - b \) which implies that revenues are linear in profitability and more dispersion should not have any effect on aggregates. Hence, the dynamics of aggregate revenues discussed are mainly driven by the dynamics of the aggregate capital stock which initially drops and subsequently rebounds and overshoots considerably. Note also that the case at hand can be seen as a maximum possible drop for TFP shocks with market power because under the assumptions of constant RTS in production with

\(^{43}\)Recall that for pure TFP shocks the revenue function is equivalent to the production function so that output also experiences a boom.
Figure 5: The impact of an uncertainty shock with adjustment costs

Notes: All the simulation results except for the lower two panels are for models where a Jensen correction is applied to the mean of the idiosyncratic profitability process. Aggregate labor demand and profits display the same dynamics as aggregate revenues and are therefore not shown separately. All values are specified in percentage deviations from the initial level.
capital and labor shares of $1/3$ and $2/3$ and a demand elasticity of only three, the implied value of $c$ is already higher than $1 - b$.

Finally, for the case of uncertainty shocks to demand, i.e. when $c = 1 - a - b$, a prolonged drop in aggregate revenues of around 1 to 1.5% is induced that lasts for five consecutive quarters until uncertainty is low again. For the C&H and the Bloom adjustment cost specifications revenues subsequently overshoot the initial level by a similar magnitude and then gradually revert back. In contrast, for the Bloom 2 adjustment cost specification the rebound is much lower so that aggregate revenues are still around 0.7% below the initial level 15 quarters after the uncertainty shock hit the system. This is due to the fact that the mean of idiosyncratic profitability drops by around 0.75% after the uncertainty shock and stays there for an extended period of time, as was shown in the top left panel of figure 2.\textsuperscript{44} For completeness, the lowest two panels in figure 5 plot the impact of an uncertainty shock to demand when no Jensen correction is applied to the mean of idiosyncratic profitability. Due to the fact that in this case average profitability now increases after an uncertainty shock, the drops in aggregate revenues are lower and of shorter duration, while the overshoots are much larger and more persistent.

In summary, it has been shown that while uncertainty shocks to demand cause the type of drop, rebound and overshoot dynamics as emphasized in Bloom (2009), uncertainty shocks to TFP lead to considerable and prolonged booms. The reason for this is that the revenue function is likely to have increasing RTS in $(A, K, L)$ space when we consider TFP shocks. This in turn leads to the fact that higher ex-post cross-sectional dispersion of TFP is a time of opportunity. In other words, the distributional effect of uncertainty shocks is highly positive.

### 4.4 Disentangling Expectational and Distributional Effects

In order to further analyze the factors driving the results presented above, it is useful to consider the magnitudes of the expectational and distributional effects of uncertainty shocks for each of the models. To this end, two similar simulation exercises as in the previous subsection are performed. The difference is that once only the expectations of firms are affected by the uncertainty shock but not the actual distribution of profitability, while the other time expectations are not affected by the uncertainty shock but the distribution of profitability across firms changes. The first simulation exercise identifies the expectational effect given in equation (15) while the second simulation identifies the distributional effect given in equation (16).

Before moving to the results it is useful to visualize the driving forces at work in each of the simulations. The top two panels of figure 6 show how expectations of firms and the dispersion of idiosyncratic profitability behave in the case of the two simulations. From the upper left panel it can be seen that in the simulation of expectational effects firms start to expect higher dispersion in future profitability shocks when the uncertainty shock hits in period zero. In contrast, firms always expect low dispersion in the simulation of distributional effects. In the upper right panel of the figure the corresponding dynamics of the standard deviation of id-

\textsuperscript{44}This is also the reason for the undershooting of aggregate revenues for the C&H and Bloom specifications towards the final quarters of the simulation.
Figure 6: The expectational and distributional effect for aggregate revenues

Notes: All the simulation results are for models where a Jensen correction is applied to the mean of the idiosyncratic profitability process. The values are specified in percentage deviations from the initial level.
iosyncratic profitability are displayed. While the dispersion of profitability does not change in the simulation of expectational effects, dispersion increases from period one to five and gradually falls back in the simulation of distributional effects. The standard simulations of an uncertainty shock from the previous subsection would feature the dynamics of expectations from the simulation of expectational effects and the dynamics of dispersion from the simulation of distributional effects.

The remaining panels in figure 6 display the expectational and distributional effects of aggregate revenues for each of the models under consideration. The main feature to note is that while the value of $c$ only has minor quantitative implications for the expectational effect of uncertainty shocks, the magnitude of the distributional effect changes considerably between uncertainty shocks to demand and TFP. To be precise, the distributional effect is an order of magnitude higher for pure TFP shocks than for demand shocks.

Quantitatively, the expectational effect is negative for all models under consideration and the maximum drops in aggregate revenues are between 0.5 and 4.5%. Moreover, the difference in the maximum drop of aggregate revenues between demand and TFP shocks is only around 1%, indicating that the expectational effect does not change much with the value of $c$. In contrast, the magnitude of the positive distributional effect is 12 to 26% for pure TFP shocks, while it is -0.5 to 3% for demand shocks. Thus, the main driver behind the results found in the previous subsection is that the distributional effect of uncertainty shocks substantially increases with the RTS in the revenue function.

5 Conclusion

This paper has looked at the question of whether temporarily higher uncertainty can cause recessions. To answer this question a partial equilibrium model with heterogeneous firms, various forms of capital adjustment costs and time-varying uncertainty was built and simulated. The main difference compared to the existing literature was that the model set-up allowed for the separate analysis of uncertainty shocks to demand and TFP.

The main finding coming out of the analysis is that while uncertainty shocks to demand lead to the drop, rebound and overshoot dynamics reminiscent of recessions, uncertainty shocks to TFP are likely to lead to considerable and prolonged booms. The main reason for these differing dynamics is that the positive distributional effect of uncertainty shocks is much larger for TFP than for demand. The intuition for this is that the revenue function is likely to have increasing RTS for TFP shocks while for demand shocks the RTS can be constant at best. Therefore, more ex-post cross-sectional dispersion in TFP is a time of opportunity for firms.

In the case of uncertainty shocks to TFP this positive distributional effect more than compensates for the negative expectational effect of uncertainty shocks that arises from real-options effects in the presence of non-convex adjustment costs, thus leading to large and persistent economic booms. In contrast, for uncertainty shocks to demand the negative expectational effect dominates until the point where uncertainty turns low again but cross-sectional dispersion is still large. Hence, uncertainty shocks to demand lead to business cycle like dynamics that feature a drop, rebound.
and overshoot as emphasized by Bloom (2009).

The results in this paper where derived under the assumptions of risk-neutral firms and constant factor prices in order to isolate the effects of uncertainty shocks that work through the presence of capital adjustment costs. Even though the papers by Bloom et al. (2010) and Bachmann and Bayer (2011) consider general equilibrium effects, such as a stochastic discount factor and varying wages, it is not clear how these factors move after an uncertainty shock and affect the response due to the presence of adjustment costs. In future research it is therefore necessary to determine the empirical responses of factor prices to uncertainty shocks and how these factor prices respond in general equilibrium models with uncertainty shocks.
Appendix A: The Demand Function

Let demand for the product of firm $i$ at time $t$ be given by the following constant elasticity demand function:

$$q_{i,t} = B_{i,t} \cdot p_{i,t}^{-\varepsilon}$$

(23)

Here $q$ denotes output, $p$ is the price, $B$ is a potentially time-varying demand shifter, and $-\varepsilon$ is the constant price elasticity of demand. From now on firm and time subscripts will be omitted unless deemed necessary. To see that the price elasticity is given by $-\varepsilon$, recall that the price elasticity of demand is defined as:

$$E_{qp} = \frac{dq}{q} \frac{1}{dp} = \frac{dq}{dp} \frac{1}{p}$$

(24)

Now, taking the derivative of equation (23) with respect to the price yields:

$$\frac{dq}{dp} = -\varepsilon B p^{-\varepsilon-1} = -\varepsilon \frac{q}{p}$$

(25)

Plugging this into the definition of the elasticity given in equation (24) yields:

$$E_{qp} = -\varepsilon \frac{q}{p} \frac{1}{q} = -\varepsilon$$

(26)

This constant elasticity demand function implies the following inverse demand function:

$$p_{i,t} = B_{i,t}^{\frac{1}{\varepsilon}} \cdot q_{i,t}^{-\frac{1}{\varepsilon}}$$

(27)

From this inverse demand function it is easy to see that as $\varepsilon \to \infty$, we approach the case of perfect competition as the price becomes completely unresponsive to the level of output supplied by the firm. In contrast, as $\varepsilon \to 0$, the price responds heavily to small changes in output. Thus, the higher the elasticity of demand $\varepsilon$, the lower the degree of market power. For this type of demand function it is important to notice that the profit maximization problem of a firm is only well defined if $\varepsilon > 1$. To see this, define revenues as $R = pq$. Remembering the fact that the price is a function of output, the change in revenues with respect to output is given by:

$$\frac{dR}{dq} = \frac{dp}{dq} q + p$$

(28)

Differentiating equation (27) with respect to output yields:

$$\frac{dp}{dq} = -\frac{p}{\varepsilon q}$$

(29)

Using this in equation (28) gives us:

$$\frac{dR}{dq} = -\frac{1}{\varepsilon q} p + p = p \left( 1 - \frac{1}{\varepsilon} \right) = B^{\frac{1}{\varepsilon}} q^{-\frac{1}{\varepsilon}} \left( 1 - \frac{1}{\varepsilon} \right)$$

(30)

Clearly, whenever $0 < \varepsilon < 1$ total revenues will be globally decreasing in output. Assuming that costs are increasing in output, this implies that the firm would like to produce as little as possible in order to maximize profits. Therefore, we require that $\varepsilon > 1$ for the problem of the firm to be well defined and nondegenerate.
Appendix B: Derivation of the Revenue Function

In the following a detailed derivation of the revenue function is provided in order to motivate the differences in functional forms that are implied by demand and supply shocks. It is assumed that firms differ in their productivity and potentially in the demand that they face. Let the production function of firm \( i \) at time \( t \) be given by:

\[
Y(\tilde{A}_{i,t}, K_{i,t}, L_{i,t}) = \tilde{A}_{i,t}K_{i,t}^{\nu}L_{i,t}^{\omega}
\]  

(31)

where \( \tilde{A} \) is total factor productivity, \( K \) is the capital stock, \( L \) is labor input, and \( \nu \) and \( \omega \) are parameters that satisfy \( \nu, \omega \in (0,1) \) and \( \nu + \omega \in (0,1] \). This specification allows for both constant and decreasing returns to scale. For example, whenever \( \nu + \omega = 1 \) we have constant returns to scale and the production function is given by the familiar Cobb-Douglas form. Demand for the output of firm \( i \) is given by the following constant elasticity demand function:

\[
q_{i,t} = B_{i,t}p_{i,t}^{-\varepsilon}
\]  

(32)

Here \( q \) denotes output demanded, \( p \) is the price, \( B \) is a potentially time-varying demand shifter, and \( -\varepsilon \) is the constant price elasticity of demand. It is required that \( \varepsilon > 1 \) for the problem of the firm to be well defined.\(^{45}\) Given this production and demand function, total revenue of each firm can be expressed as:

\[
\tilde{S}(B_{i,t}, \tilde{A}_{i,t}, K_{i,t}, L_{i,t}) = p_{i,t}Y_{i,t} = B_{i,t}^{-1}Y_{i,t}^{-\frac{1}{\varepsilon}}Y_{i,t} = B_{i,t}^{-1}Y(\tilde{A}_{i,t}, K_{i,t}, L_{i,t})^{\frac{\varepsilon-1}{\varepsilon}}
\]  

(33)

Now let us define the parameters \( a = \nu^{\frac{\varepsilon-1}{\varepsilon}} \) and \( b = \omega^{\frac{\varepsilon-1}{\varepsilon}} \). Moreover, we can combine the effects of total factor productivity and demand into one auxiliary variable. This is achieved by defining a shock to revenues as \( \tilde{A}_{i,t}^{c} = B_{i,t}^{1/\varepsilon} \tilde{A}_{i,t}^{(\varepsilon-1)/\varepsilon} \). With these transformations we can represent the revenue function of the firm as:\(^{46}\)

\[
S(A_{i,t}, K_{i,t}, L_{i,t}) = A_{i,t}^{c}K_{i,t}^{a}L_{i,t}^{b}
\]  

(34)

\(^{45}\)See Appendix A for a discussion of the properties of this kind of demand function.

\(^{46}\)The derivation of this revenue function is closely related to the one in Bloom (2009) with the difference that in his model he assumes \( c = 1 - a - b \), while the specification in this paper allows for flexible values of \( c \).
Appendix C: Expectations with Log-Normal Profitability

To isolate the effects of time-varying uncertainty, we want an increase in uncertainty to correspond to a mean preserving spread of the relevant variable in question. Therefore, as uncertainty is equivalent to the variance of shocks in our model, we want that a change in the variance of shocks has no effect on the expected value of profitability. If the AR(1) process for idiosyncratic profitability is specified in levels, a change in the variance of shocks naturally corresponds to a mean preserving spread. However, if the AR(1) process is specified in logs, then a change in the variance of shocks will also have an effect on the expected value of the variable in levels. In other words, when we specify the AR(1) process for idiosyncratic profitability in logs, a change in the variance of shocks does not correspond to a mean preserving spread. To see this analytically, take the AR(1) process in logs for idiosyncratic profitability from equation (7) and transform it into levels:

\[ \psi_{i,t} = \mu_{\psi} + \rho_{\psi} \psi_{i,t-1} + \nu_{i,t} \]  
\[ \Leftrightarrow e^{\psi_{i,t}} = e^{(\mu_{\psi} + \rho_{\psi} \psi_{i,t-1} + \nu_{i,t})} \]  
\[ \Leftrightarrow \Psi_{i,t} = e^{\mu_{\psi}} \psi_{i,t-1} \nu_{i,t} \]  

If we now take the expectation of the variable in levels and apply the fact that for a normally distributed variable, \( \nu_{i,t} \sim N(\mu_{\nu,t-1}, \sigma_{\nu,t-1}^2) \), we have \( E[e^{-a\nu_{i,t}}] = e^{-a\mu_{\nu,t-1} + a^2\sigma_{\nu,t-1}^2/2} \) we get the following result:

\[ E[\Psi_{i,t}] = e^{\mu_{\psi}} \psi_{i,t-1} E[e^{\nu_{i,t}}] \]  
\[ \Leftrightarrow E[\Psi_{i,t}] = e^{\mu_{\psi}} \psi_{i,t-1} e^{\mu_{\nu,t-1} + \sigma_{\nu,t-1}^2/2} \]  

It is obvious from this expression that the expected value of idiosyncratic profitability in levels is increasing in the variance of shocks unless the mean of the shocks changes in an offsetting manner with the variance. A natural way to avoid that the expected value of profitability rises in response to an increase in the shock variance is therefore to specify the mean of the shocks as \( \mu_{\nu,t-1} = -\sigma_{\nu,t-1}^2/2 \). This adjustment is labeled as a Jensen correction throughout the paper, as the increase in the expected value in levels is a result of Jensen’s Inequality.
Appendix D: The Capital Policy Function without Adjustment Costs

In the absence of capital adjustment costs and irreversibilities, the firm’s decision problem can be represented by the following Bellman Equation which is a special case of equation (11):

\[ V(A, K, \sigma_v) = \max_{K'} \Pi(A, K) - p(K' - K(1 - \delta)) + \beta E_{A', \sigma_v|A, \sigma_v}[V(A', K', \sigma_v')] \] (40)

Assuming that \( V(A, K, \sigma_v) \) is differentiable, the first-order condition with respect to next period capital is:

\[ p = \beta E_{A', \sigma_v|A, \sigma_v}[V_{K'}(A', K', \sigma_v')] \] (41)

By the envelope condition we have that the marginal value of capital is given by:

\[ V_K(A, K, \sigma_v) = \Pi_K(A, K) + (1 - \delta)p \] (42)

Taking this condition forward one period and using it in the first-order condition yields:

\[ p = \beta E_{A', \sigma_v|A, \sigma_v}[\Pi_{K'}(A', K') + (1 - \delta)p] \] (43)

This equation states that an optimal capital choice equates the marginal cost of investment, which is given by \( p \), to the discounted expected marginal gain, which is given by marginal profits plus the market value of the depreciated increment of investment. Using the specific functional forms assumed in the paper, we can rewrite this as:

\[ p = \beta E_{A'|A, \sigma_v} \left[ \phi \frac{a}{1 - b} A'^{e_\tau - 1} K'^{\sigma_{v'}} - (1 - \delta)p \right] \] (44)

Solving for \( K' \), we arrive at the following explicit capital policy function:

\[ K'(A, \sigma_v) = \varphi E \left[ A'^{e_\tau} | A, \sigma_v \right]^{1-b}_{1-a-b} \] (45)

where \( \varphi = [a(\beta \phi)/[(1-b)[p-p(1-\delta)]]]^{1-b}_{1-a-b} \) is a constant parameter. Given the stochastic processes assumed for profitability we can derive an analytical expression for the expectation in equation (45). First, given the multiplicative specification of profitability and the AR(1) structure in logs we can derive the following expression:

\[ K' = \varphi E \left[ (Z'\Psi')^{e_\tau} | Z', \Psi, \sigma_v \right]^{1-b}_{1-a-b} \] (46)

\[ \Leftrightarrow K' = \varphi E \left[ (e^{\mu_Z} e^{\psi} Z'^{\rho_\sigma} \cdot e^{\mu_\Psi} e^{\psi'} \Psi'^{\rho_\sigma})^{e_\tau} | \sigma_v \right]^{1-b}_{1-a-b} \] (47)

\[ \Leftrightarrow K' = \varphi E \left[ (e^{\mu_Z} Z'^{\rho_\sigma} e^{\mu_\Psi} \Psi'^{\rho_\sigma})^{e_\tau} | \sigma_v \right]^{1-b}_{1-a-b} \] (48)

Making use of the fact that aggregate and idiosyncratic profitability shocks are independent of each other, we can separate the two terms within the expectations operator:

\[ K' = \varphi E \left[ (e^{\mu_Z} Z'^{\rho_\sigma} e^{\mu_\Psi} \Psi'^{\rho_\sigma})^{e_\tau} | \sigma_v \right]^{1-b}_{1-a-b} \] (49)
Furthermore, we can use the fact that for a normally distributed variable, $\epsilon \sim N(\mu, \sigma^2)$, we have $E[e^{-a\epsilon}] = e^{-a\mu + a^2\sigma^2/2}$:

$$K' = \varphi(e^{\mu_{t-1}Z_{t-1}}e^{\mu_{t-1}}g_{t-1}e^{\mu_{t-1}}g_{t-1}e^{\mu_{t-1}}g_{t-1})^{\frac{1-a}{1-a}}$$ (50)

$$\Leftrightarrow K' = \varphi(e^{\mu_{t-1}Z_{t-1}}e^{\mu_{t-1}}g_{t-1}e^{\mu_{t-1}}g_{t-1}e^{\mu_{t-1}}g_{t-1})^{\frac{1-a}{1-a}}$$ (51)

From this explicit capital policy function for the case without adjustment frictions we can learn some useful insights. First, unless the mean of idiosyncratic shocks is changing at the same time as the variance, the desired capital stock will be increasing in the variance of idiosyncratic shocks, independent of the value of $c$. This is simply a result of the fact that the expected value of profitability is increasing in the variance of idiosyncratic shocks given the assumed AR(1) process in logs. 47 We can adjust for this effect by letting $\mu_{t-1} = -\sigma_{t-1}^2/2$, so that the expected value of profitability does not change with the variance of shocks. With this structure equation (51) can be rewritten as:

$$K' = \varphi(e^{\mu_{t-1}Z_{t-1}}e^{\mu_{t-1}}g_{t-1}e^{\mu_{t-1}}g_{t-1}e^{\mu_{t-1}}g_{t-1})^{\frac{1-a}{1-a}}e^{\frac{\sigma_{t-1}^2(1-b)}{(1-a)(1-a)}}$$ (52)

47 A derivation of this result can be found in Appendix C.
Appendix E: Dynamics of the Profitability Distribution

The distribution of idiosyncratic profitability across firms is log-normal, given the AR(1) process in logs assumed in equation (7). Applying the expectations and variance operator to this equation, the dynamics of the cross-sectional mean and variance of idiosyncratic profitability in logs can be expressed as follows:

\[ E[\psi_{i,t}] = E[\mu_\psi + \rho_\psi \psi_{i,t-1} + \psi_{i,t}] \]  
\[ \Leftrightarrow E[\psi_{i,t}] = \mu_\psi + \rho_\psi E[\psi_{i,t-1}] + E[\psi_{i,t}] \]  
\[ \Leftrightarrow E[\psi_{i,t}] = \mu_\psi + \rho_\psi E[\psi_{i,t-1}] + \mu_{v,t-1} \]  
\[ V[\psi_{i,t}] = V[\mu_\psi + \rho_\psi \psi_{i,t-1} + \psi_{i,t}] \]  
\[ \Leftrightarrow V[\psi_{i,t}] = \rho_\psi^2 V[\psi_{i,t-1}] + V[\psi_{i,t}] \]  
\[ \Leftrightarrow V[\psi_{i,t}] = \rho_\psi^2 V[\psi_{i,t-1}] + \sigma_{v,t-1}^2 \]  

If we assume that the mean and variance of idiosyncratic shocks have been in the low state for the infinite past up to and including period \( t \), these two moments become:

\[ E[\psi_{i,t}] = \frac{\mu_\psi + \mu^{L}_v}{1 - \rho_\psi} \]  
\[ V[\psi_{i,t}] = \frac{(\sigma^{L}_v)^2}{1 - \rho_\psi^2} \]  

Now let us define the variable \( \Delta \mu_{v,t} = \mu_{v,t} - \mu^{L}_v \), which captures the difference between the mean of idiosyncratic shocks under the current regime and the low regime. Analogous to this let us define the variable \( \Delta \sigma_{v,t}^2 = \sigma_{v,t}^2 - (\sigma^{L}_v)^2 \). Starting the evolution of the mean of log idiosyncratic profitability from the value in equation (59) we can express the dynamics as follows:

\[ E[\psi_{i,t+1}] = \frac{\mu_\psi + \mu^{L}_v}{1 - \rho_\psi} + \Delta \mu_{v,t} \]  
\[ E[\psi_{i,t+2}] = \frac{\mu_\psi + \mu^{L}_v}{1 - \rho_\psi} + \Delta \mu_{v,t+1} + \rho_\psi \Delta \mu_{v,t} \]  
\[ E[\psi_{i,t+3}] = \frac{\mu_\psi + \mu^{L}_v}{1 - \rho_\psi} + \Delta \mu_{v,t+2} + \rho_\psi \Delta \mu_{v,t+1} + \rho_\psi^2 \Delta \mu_{v,t} \]  

Similarly, starting from the variance in equation (60), the evolution of the vari-
ance of log idiosyncratic profitability can be expressed as:

\[
V[\psi_{i,t+1}] = \frac{(\sigma_{\psi}^L)^2}{1 - \rho_{\psi}^2} + \Delta \sigma_{v,t}^2 \tag{64}
\]

\[
V[\psi_{i,t+2}] = \frac{(\sigma_{\psi}^L)^2}{1 - \rho_{\psi}^2} + \Delta \sigma_{v,t+1}^2 + \rho_{\psi}^2 \Delta \sigma_{v,t}^2 \tag{65}
\]

\[
V[\psi_{i,t+3}] = \frac{(\sigma_{\psi}^L)^2}{1 - \rho_{\psi}^2} + \Delta \sigma_{v,t+2}^2 + \rho_{\psi}^2 \Delta \sigma_{v,t+1}^2 + \rho_{\psi}^4 \Delta \sigma_{v,t}^2 \tag{66}
\]

\[\vdots\]

From these recursions, it is easy to see that we can express the mean and variance of log idiosyncratic profitability as the sum of the value that would prevail under the low regime and the accumulated effect due to occasional switches to the high regime:

\[
E[\psi_{i,t}] = \frac{\mu_{\psi} + \mu_{\psi}^L}{1 - \rho_{\psi}} + \sum_{j=1}^{\infty} \rho_{\psi}^{j-1} \Delta \mu_{v,t-j} \tag{67}
\]

\[
V[\psi_{i,t}] = \frac{(\sigma_{\psi}^L)^2}{1 - \rho_{\psi}^2} + \sum_{j=1}^{\infty} \rho_{\psi}^{2(j-1)} \Delta \sigma_{v,t-j}^2 \tag{68}
\]

In the end however, we are interested in the evolution of the distribution of idiosyncratic profitability in levels. Given that log idiosyncratic profitability is normally distributed, the mean and variance of idiosyncratic profitability in levels can be derived along the following steps:

\[
E[\Psi_{i,t}] = E[e^{\log(\Psi_{i,t})}] = E[e^{\psi_{i,t}}] \tag{69}
\]

\[
\Leftrightarrow E[\Psi_{i,t}] = e^{E[\psi_{i,t}]+V[\psi_{i,t}]/2} \tag{70}
\]

\[
V[\Psi_{i,t}] = E[(\Psi_{i,t} - E[\Psi_{i,t}])^2] \tag{71}
\]

\[
\Leftrightarrow V[\Psi_{i,t}] = E[\Psi_{i,t}^2] - 2E[\Psi_{i,t}]^2 + E[\Psi_{i,t}]^2 \tag{72}
\]

\[
\Leftrightarrow V[\Psi_{i,t}] = E[\Psi_{i,t}^2] - E[\Psi_{i,t}]^2 \tag{73}
\]

\[
\Leftrightarrow V[\Psi_{i,t}] = E[e^{2\log(\Psi_{i,t})}] - E[e^{\log(\Psi_{i,t})}]^2 \tag{74}
\]

\[
\Leftrightarrow V[\Psi_{i,t}] = e^{2E[\psi_{i,t}]+2V[\psi_{i,t}]} - (e^{E[\psi_{i,t}]+V[\psi_{i,t}]/2})^2 \tag{75}
\]

\[
\Leftrightarrow V[\Psi_{i,t}] = e^{2E[\psi_{i,t}]+2V[\psi_{i,t}]} - e^{2E[\psi_{i,t}]+V[\psi_{i,t}]} \tag{76}
\]
Appendix F: Numerical Solution Technique and Accuracy

The model described in section 2 does not have a closed-form solution once we allow for the various forms of adjustment costs and partial irreversibilities. In the remaining parts of the paper, the model is therefore solved numerically by discrete value function iteration in order to study the properties of investment policy functions and the models' implications for the effect of uncertainty shocks. For this procedure to be feasible, the profitability process needs to be approximated by a discrete markov chain. This approximation is done using the method proposed by Tauchen (1986), adapted to a markov-switching process. The grids for the value function iteration are chosen with 21 idiosyncratic profitability points, 10 aggregate profitability points, 2 uncertainty states, and between 350 and 1000 capital grid points depending on the model.\footnote{It was checked that the results are not sensitive to an increase in the number of grid points.} At this point it is useful to asses the accuracy of the numerical approximation that is employed. For this purpose two exercises are performed. First, the dynamics of the discretized process for idiosyncratic profitability are compared to the true dynamics of the AR(1) process in logs. Second, the dynamics of the approximated model without adjustment costs are compared to the analytical results derived above.

As mentioned above the profitability process is approximated by a discrete markov chain with 21 grid points. The question is of course whether this discretized process resembles the original AR(1) process in logs in important dimensions. To answer this question, the dynamics of the cross-sectional mean and standard deviation after an uncertainty shock are compared between the discretized process for idiosyncratic profitability and it's analytical counterpart. To this end a sample of 250,000 units is simulated for 25 periods. The initial distribution of units is set to the unconditional distribution under the low uncertainty regime. It is then assumed that in period zero an uncertainty shock hits the system lasting for 5 periods. For all subsequent periods uncertainty is set to the low regime again. Figure 7 displays the results of this exercise for the cases with and without a Jensen correction applied to the mean of shocks.

Regarding the accuracy of the discretized process for idiosyncratic profitability it can be observed that the dynamics of the mean and standard deviation are replicated fairly accurately in both cases. The standard deviation of idiosyncratic profitability increases considerably for five consecutive periods, and then falls back gradually as uncertainty becomes low again. The decrease in the mean with the Jensen correction and the increase in the mean without the Jensen correction are also captured fairly accurately by the discretized process. However, both the mean and standard deviation are consistently higher than for the true process. In the case of the mean this difference is approximately 1%, while for the standard deviation this difference is around 14%. Nevertheless, the accuracy of the approximated profitability process is deemed sufficient as it replicates the dynamics of the first two moments after an uncertainty shock fairly well.

We can therefore turn to the accuracy of the aggregate dynamics produced by
Figure 7: Moments of the idiosyncratic profitability distribution

Figure 8: Accuracy of the approximation method without adjustment costs
a discretized model without any adjustment costs. For this model we can use the analytical results developed in section 3 as a benchmark to compare to. The parameters are set to the values discussed in section 4.1 and shown in table 3. In addition, all adjustment costs are switched off and $c$ is set to $1 - a - b$. Similar to the previous simulation exercise, a panel of 1,000,000 units is simulated for 25 periods. In period zero an uncertainty shock occurs for five consecutive periods and uncertainty is assumed to be low for all other periods before and after.

Figure 8 shows the results of this simulation. Looking at the means of idiosyncratic profitability, the capital stock, investment and revenues, profits and labor we see that an uncertainty shock leads to a prolonged drop in all variables. It is straightforward to see that the dynamics produced by the numerical approximation closely resemble the dynamics that are obtained from the analytical formulas or a simulation of the analytical model. The deviations that do occur are partly due to sampling and otherwise deemed sufficiently small. We can therefore safely move on to analyze the simulation results for more complicated models with adjustment costs.
Appendix G: Existing Adjustment Cost Estimates

There are two main papers that estimate a rich set of capital adjustment costs for models similar to the one in section 2. In the following their results and differences are briefly outlined.

Cooper and Haltiwanger (2006) estimate capital adjustment costs using simulated method of moments (SMM), disregarding labor adjustment costs. They use annual data for around 7,000 large continuing manufacturing plants between 1972 and 1988 from the Longitudinal Research Database (LRD) and attempt to match the following four moments with their model: the fraction of investment bursts, the fraction of investment falls, the investment autocorrelation, and the correlation of investment with profitability. Their model is specified at an annual frequency so there is no time aggregation and they estimate the profitability process and revenue function together with the adjustment costs. Their estimated adjustment cost parameters can be found in table 4. One caveat of their estimation is that even though they report that the inaction rate in the data is 8.1%, they do not try to match this moment and a model with their estimated parameter values actually leads to an inaction rate of over 80%.

Bloom (2009) jointly estimates capital and labor adjustment costs using SMM. The data he uses is a panel of 2,548 publicly traded U.S. firms from Compustat spanning the years 1981 to 2000. The firms are large (at least 500 employees and $10m sales) and span all sectors of the economy. The model he uses is specified at a monthly frequency and at the plant level so that there is aggregation across time as well as across units in order to correspond to the data which is annual and at the firm level. The moments that he attempts to match with the model are the dynamic auto- and cross-correlations as well as the standard deviation and skewness of investment rates, employment growth rates and sales growth rates. The author does not estimate the revenue function and only estimates the variance of the profitability process. The estimated adjustment cost parameters can also be found in table 4.

To summarize the existing estimates, Cooper and Haltiwanger (2006) find considerable fixed costs of investment and only small irreversibilities and convex adjustment costs, while Bloom (2009) finds large irreversibilities and small convex and non-convex adjustment costs. These differences in the estimates can be due to the following reasons:

- **Data:** manufacturing vs. all industries, plant vs. firm, 72-88 vs. 81-00
- **Model:** annual vs. monthly, specification of driving process different
- **Moments:** different moments they try to match

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49 Investment here always refers to the investment rate relative to the capital stock. Bursts and spikes are defined as investment rates exceeding plus or minus 20%.

50 The profitability process is assumed to follow a geometric random walk, and the variance is assumed to be the same for the plant, firm, and aggregate component.

51 Cooper and Haltiwanger (2006) estimate an AR(1) process in logs, while Bloom (2009) assumes a random walk in levels.
References


